MOSFETS | Small Signal Model is the same for N and P mos

$V_P = \sqrt{2}\, V_{RMS}$

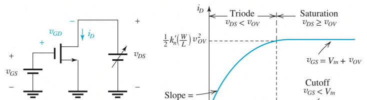
Fundamental Formulas

look for Formulas that Solve for the same Values

$V_{OV} = V_{GS} - V_t$

$I_D = \frac{1}{2} K_x \left(\frac{W}{L}\right)(V_{OV})^2 \hookrightarrow K_x = \mu_x C_{ox}$

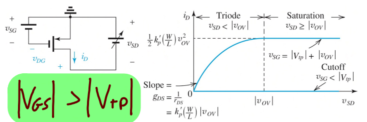
$\frac{I_x}{I_y} = \frac{(W/L)_x}{(W/L)_y}$ → Only when Gate is Shorted to S/D



$V_{GS} > V_{tn}$

- $v_{GS} < V_{tn}$: no channel; transistor in cutoff; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$: a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched off at the drain end;

Triode Region	Saturation Region
Continuous channel, obtained by: $v_{GD} > V_{tn}$ or equivalently: $v_{DS} < v_{OV}$	Pinched-off channel, obtained by: $v_{GD} \leq V_{tn}$ or equivalently: $v_{DS} \geq v_{OV}$
Then, $i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS}-V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$	Then $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS}-V_{tn})^2$
or equivalently, $i_D = k_n'\left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$	or equivalently, $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)v_{OV}^2$

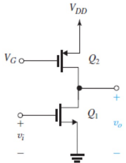


$|V_{GS}| > |V_{tp}|$

- $v_{SG} < |V_{tp}|$: no channel; transistor in cutoff; $i_D = 0$
- $v_{SG} = |V_{tp}| + |v_{OV}|$: a channel is induced; transistor operates in the triode region or in the saturation region depending on whether the channel is continuous or pinched off at the drain end;

Triode Region	Saturation Region
Continuous channel, obtained by: $v_{DG} > |V_{tp}|$ or equivalently $v_{SD} < |v_{OV}|$	Pinched-off channel, obtained by: $v_{DG} \leq |V_{tp}|$ or equivalently $v_{SD} \geq |v_{OV}|$
Then $i_D = k_p'\left(\frac{W}{L}\right)\left[(v_{SG}-|V_{tp}|)v_{SD} - \frac{1}{2}v_{SD}^2\right]$	Then $i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)(v_{SG}-|V_{tp}|)^2$
or equivalently $i_D = k_p'\left(\frac{W}{L}\right)\left(|v_{OV}| - \frac{1}{2}v_{SD}\right)v_{SD}$	or equivalently $i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)v_{OV}^2$

CMOS



The circuit in **Fig. A3** is fabricated in the **0.18-µm CMOS** process whose parameters are specified in **Table K.1 (slide 1)**. The supply voltage V_{DD} = 1.8 V. Design the circuit to obtain a voltage gain A_v = −20 V/V. Use devices of equal length L operating at a drain current I=100 µA and $|V_{OV}|$ = 0.2 V. Determine the required values of V_G, L, $(W/L)_1$, and $(W/L)_2$.

$V_G = V_{DD} - V_{SG_2} = 1.8 - 0.5 - 0.2 = 1.1V$

$(W/L)_2 = \frac{2 \cdot I_{D_2}}{K_n \cdot |V_{OV}^2|} = \frac{2 \cdot 100\mu}{86\mu \cdot 0.2^2} = 58.14$

$(W/L)_1 = \frac{2 \cdot 100\mu}{387\mu \cdot 0.2^2} = 12.92$

$A_v = -g_{m_1}(r_{o_1} \| r_{o_2})$ $g_{m_1} = \frac{2 \cdot 100\mu}{0.2} = 1mA/V$

→ $r_{o_1} \| r_{o_2} = -\frac{A_v}{g_{m_1}} = \frac{20}{1m} = 20k\Omega = \frac{V_{An} \cdot L}{I} \| \frac{V_{Ap} \cdot L}{I} = 50L \| 60L = 27.3L\ k\Omega$

→ $L = \frac{20}{27.3} = 0.73\mu m$

$K_n = 387$
$K_p = 86$
$V_T = \pm 0.5V$

Other Formulas

$|E| = \frac{V_{DS}}{L}$

μ_x = mobility of holes/e^-

	Characteristics				
Amplifier type	R_{in}	A_{vo}	R_o	A_v	G_v
Common source (Fig. 7.36)	∞	$-g_m R_D$	R_D	$-g_m(R_D \| R_L)$	$-g_m(R_D \| R_L)$
Common source with R_s (Fig. 7.38)	∞	$-\frac{g_m R_D}{1+g_m R_s}$	R_D	$\frac{-g_m(R_D \| R_L)}{1+g_m R_s}$ $-\frac{R_D \| R_L}{1/g_m + R_s}$	$-\frac{g_m(R_D \| R_L)}{1+g_m R_s}$ $-\frac{R_D \| R_L}{1/g_m + R_s}$
Common gate (Fig. 7.40)	$\frac{1}{g_m}$	$g_m R_D$	R_D	$g_m(R_D \| R_L)$	$\frac{R_D \| R_L}{R_{sig} + 1/g_m}$
Source follower (Fig. 7.43)	∞	1	$\frac{1}{g_m}$	$\frac{R_L}{R_L + 1/g_m}$	$\frac{R_L}{R_L + 1/g_m}$
General:	$\frac{V_i}{I_i}$	$\frac{V_o}{V_i}$		$\frac{V_o}{V_i}$	$\frac{V_o}{V_{sig}}$

Amp - Summary table

Small-Signal Parameters

NMOS transistors

- **Transconductance:**

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{OV}}$$

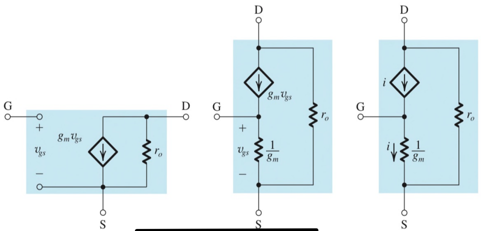
- Output resistance:

$$r_o = V_A / I_D = 1/\lambda I_D = \frac{1}{g_{ds}} \rightarrow g_{ds} = \frac{i_{DS}}{V_{DS}}$$

PMOS transistors

Same formulas as for NMOS *except* using $|V_{OV}|$, $|V_A|$, $|\lambda|$ and replacing μ_n with μ_p.

Small-Signal, Equivalent-Circuit Models



More Formulas

$V_A = \frac{1}{\lambda}$ / $r_o = \frac{1}{\lambda I_D}$

χ ⇒ used for Body Effect

$\hookrightarrow g_{mb} = \chi g_m$

$\hookrightarrow A_v = \frac{1}{1+\chi}$

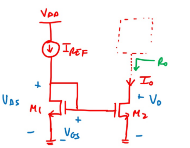
$\gamma \cong 0.4\sqrt{V}$

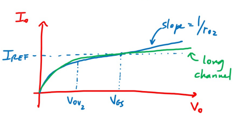
Cascode Amp's

Case	R_L	R_{in2}	R_{d1}	A_{v1}	A_{v2}	A_v
1	∞	∞	r_o	$-g_m r_o$	$g_m r_o$	$-(g_m r_o)^2$
2	$(g_m r_o) r_o$	r_o	$r_o/2$	$-\frac{1}{2}(g_m r_o)$	$g_m r_o$	$-\frac{1}{2}(g_m r_o)^2$
3	r_o	$\frac{2}{g_m}$	$\frac{2}{g_m}$	-2	$\frac{1}{2}(g_m r_o)$	$-(g_m r_o)$
4	$0.2r_0$	$\frac{1.2}{g_m}$	$\frac{1.2}{g_m}$	-1.2	$0.17(g_m r_o)$	$-0.2(g_m r_o)$

Current mirrors

CURRENT MIRROR with CURRENT SOURCE I_{REF}





M_2 in SAT → high r_o

M_1 in SAT → diode connected

For M_2 to be in SAT

$V_o \geqslant V_{GS} - V_t = V_{OV_2} = \sqrt{\frac{2 I_{D_2}}{K_n (W/L)_2}}$ ←

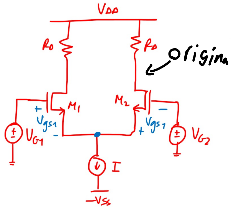
output resistance $R_o = r_{o_2}$

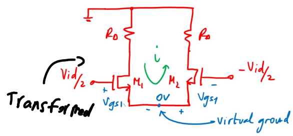
$I_o = I_{REF}$ for $V_o = V_{GS}$

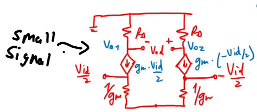
$$\begin{cases} I_{REF} = \frac{1}{2} K_n \left(\frac{W}{L}\right)_1 V_{OV_1}^2 (1 + \lambda V_{DS_1}) \\ I_o = \frac{1}{2} K_n (W/L)_2 V_{OV_2}^2 (1 + \lambda V_o) \end{cases}$$

$\frac{I_o}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1} \cdot \frac{(1+\lambda V_o)}{(1+\lambda V_{DS_1})}$ $\lambda \propto 1/L$

Differential Amps sample







$$\begin{cases} V_{G1} = \frac{1}{2}V_{id} + V_{CM} \\ V_{G2} = -\frac{1}{2}V_{id} + V_{CM} \end{cases}$$

Gain

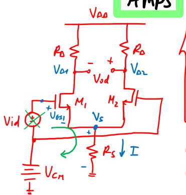
$$\left.\begin{aligned} V_{o1} &= -g_m \frac{V_{id}}{2} \cdot R_D \\ V_{o2} &= -\left(-g_m \frac{V_{id}}{2}\right) \cdot R_D \end{aligned}\right\} \quad V_{o2} - V_{o1} = \left[g_m \frac{V_{id}}{2} \cdot R_D - \left(-g_m \frac{V_{id}}{2} \cdot R_D\right)\right]$$

$= g_m V_{id} \cdot R_D$

$A_{vd} = \frac{V_{od}}{V_{id}} = \frac{V_{o2} - V_{o1}}{V_{id}} = g_m \cdot R_D$

EXERCISE

Diff Amps



$I = 20\mu A$
$R_S = 20k\Omega$
$V_{DD} = 2V$
$K_n(W/L) = 1mA/V^2$
$V_t = 0.4V$
$\lambda = 0,\ \gamma = 0$

(a) V_{CM}
(b) R_D for $A_d = 15\ V/V$
(c) V_{D1} & V_{D2}
(d) $V_{o1}/V_{cm} = A_{cm,SE}$
(e) CMRR for 10% mismatch in R_D

(a) $V_{id} = 0$

$V_{CM} = V_{GS1} + V_S = V_{GS1} + I \cdot R_S$

$V_{GS1} = V_t + \underbrace{V_{ov}}_{V_{ov_1}} = V_t + \sqrt{\dfrac{I}{K_n(W/L)}} = 0.4 + \sqrt{\dfrac{20\mu}{1m}} = 0.54V$

$V_{CM} = 0.54 + 20\mu \cdot 20K = 0.94V$

(b) $A_d = g_m \cdot R_D$

$g_m = \dfrac{2I_D}{V_{ov}} = \dfrac{20\mu}{0.14} = 140\mu A/V$

$R_D = \dfrac{15}{140\mu} = 107k\Omega$

(c) $V_{D1} = V_{D2} = V_{DD} - I_{D1,2} \cdot R_D = 2 - 10\mu \cdot 107K = 0.93V$

(d) $\dfrac{V_{o1}}{V_{cm}} = -\dfrac{R_D}{1/g_{m1} + 2R_S} = \dfrac{107K}{\dfrac{1}{140\mu} + 40K} = -2.26\ V/V$ $\qquad R_D > 2R_S$

(e) $R_{D1} = 107k\Omega$
$R_{D2} = 107K + 10.7K = 118k\Omega$

$A_{cm} = -\dfrac{\Delta R}{2R_S} = -\dfrac{10.7K}{40K} = -0.27\ V/V$

$CMRR = \dfrac{15}{0.27} = 55.5 \rightarrow 35dB$

A4.10

Design the circuit in **Fig. 10** to obtain $I = 1\ \mu A$, $I_D = 0.5$ mA, $V_S = 2$ V, and $V_D = 5$ V. The NMOS transistor has $V_t = 0.5$ V, $k_n = 4$ mA/V^2, and $\lambda = 0$.

$K_n(W/L)$

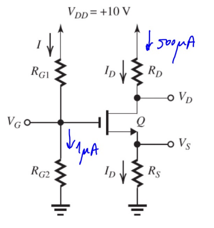


Fig. 10

$V_S = I_D \cdot R_S \rightarrow R_S = \dfrac{V_S}{I_D} = \dfrac{2}{0.5m} = 4k\Omega$

$V_D = V_{DD} - I_D R_D \rightarrow R_D = \dfrac{10-5}{0.5m} = 10k\Omega$

Assume SAT

$V_{GS} = V_t + \sqrt{\dfrac{2I_D}{K_n(W/L)}} = 0.5 + \sqrt{\dfrac{2 \cdot 0.5m}{4m}} = 1V$

$V_{ov} = 1 - 0.5 = 0.5V < V_{DS} = 3V$ SAT ✓

$V_G = V_S + V_{GS} = 2 + 1 = 3V$

$R_{G2} = \dfrac{V_G}{I} = \dfrac{3}{1\mu} = 3M\Omega$

$R_{G1} = \dfrac{V_{DD} - V_G}{I} = \dfrac{10-3}{1\mu} = 7M\Omega$

A6.5

Design the circuit in **Fig. 5** to obtain a dc voltage of 0 V at each of the drains of Q_1 and Q_2 when $v_{G1} = v_{G2} = 0$ V. Operate all transistors at $V_{OV} = 0.15$ V and assume that $V_{tn} = 0.35$ V and $K_n = 400\ \mu A/V2$. Neglect channel-length modulation.

Determine the values of R, R_D, and the W/L ratios of Q_1, Q_2, Q_3, and Q_4. What is the input common-mode voltage range for your design?

ICMR
↳ Range for which Mosfets Stay in Sat

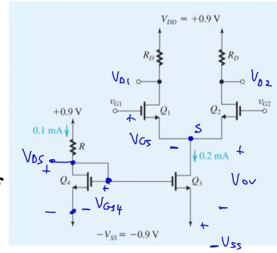


Fig. 5

A6.5 To have $V_{D1} = V_{D2} = 0V$ the voltage drop across R_D must be 0.9V

$R_D = \dfrac{0.9}{I_1} = \dfrac{2 \cdot 0.9}{0.2m} = 9k\Omega$

$(W/L)_{1,2} = \dfrac{2I_1}{K_n \cdot V_{ov}^2} = \dfrac{0.2m}{400\mu \cdot 0.15^2} = 22.2$

$(W/L)_3 = \dfrac{2I_3}{K_n V_{ov}^2} = \dfrac{2 \cdot 0.2m}{400\mu \cdot 0.15^2} = 44.4$ $\quad$ M_3 carries 2x the current of M_1 & M_2

$(W/L)_4 = (W/L)_3/2 = 22.2 \leftarrow$ M_4 carries half the current of M_3

KVL around Q_4

$V_{DD} - I_4 R - V_{DS4} - (-V_{SS}) = 0$

$V_{DS4} = V_{GS4} = V_{ov} + V_t = 0.15 + 0.35 = 0.5V$

$R = \dfrac{0.9 - 0.5 - (-0.9)}{0.1m} = 13k\Omega$

ICMR
- $V_{CM,min}$ to keep M_3 in saturation

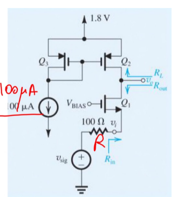
$V_{CM,min} = V_{GS1} + V_{ov} - (-V_{SS}) = V_t + 2V_{ov} - (-V_{SS}) = -0.25V$

- $V_{CM,max}$ to keep M_1 & M_2 in saturation

$V_{D1} - \cancel{V_S} > V_{CM} - \cancel{V_S} - V_t$

$V_{CM,min} = V_{D1} + V_t = 0 + 0.35V = 0.35V$

V_{ICM}



In the common-gate amplifier circuit of **Fig. A8**, Q_2 and Q_3 are matched. $K_n(W/L)_n = K_p(W/L)_p = 8$ mA/V^2 and all transistors have $|V_t| = 0.5$ V and $|V_A| = 5$ V.

The signal v_{sig} is a small sinusoidal signal with no dc component.

(a) Neglecting the effect of V_A, find the dc drain current of Q_1 and the required value of V_{BIAS}.

(b) Find the values of g_{m1} and the r_o for all transistors.

(c) Find the values of R_{in} and R_{out}

(e) Calculate the voltage gains v_o/v_i and v_o/v_{sig}.

A5.8

(a) $I_1 = I_2 = I_3 = 100\mu A$

$V_{BIAS} = V_{GS1} + I \cdot R = 0.5 + \underbrace{\sqrt{\dfrac{2 \cdot 100\mu}{8m}}}_{V_{ov_1} = 0.16V} + 100\mu \cdot 100 = 0.67V$

(b) $g_{m1} = \dfrac{2 \cdot 100\mu}{0.16} = 1.25mA/V$

All transistors will have the same r_o (why?)

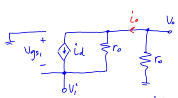
$r_{o1,2,3} = \dfrac{|V_A|}{I_D} = \dfrac{5}{100\mu} = 50k\Omega$

(c) Small-signal eq. model to determine R_{in} & R_o (See Lecture 17 & A5.5)

$R_{in} = \dfrac{1 + r_{o2}/r_{o1}}{g_{m1} + 1/r_{o1}} \approx \dfrac{2}{g_{m1}} = 1.6k\Omega$ $\quad$ since $r_{o2} = r_{o1}$ & $1/r_{o1} \approx 0$

$R_o = r_o + R + g_m r_o R = 50K + 100 + 1.25m \cdot 50K \cdot 100 = 56.4k\Omega$

(e)



$V_o = -i_o \cdot r_o$

$i_o = g_{m1} V_{gs1} + \dfrac{V_o - V_i}{r_o}$

$V_{gs1} = -V_i$

$i_o = -\dfrac{V_o}{r_o} = -g_{m1} V_i + \dfrac{V_o}{r_o} - \dfrac{V_i}{r_o}$

$\left(g_{m1} + \dfrac{1}{r_o}\right) V_i = \dfrac{2V_o}{r_o}$

$\rightarrow A_v = \dfrac{V_o}{V_i} = \dfrac{g_{m1} r_o + 1}{2} = \dfrac{1.25m \cdot 50K + 1}{2} = 31.75\ V/V$

$G_v = \dfrac{V_o}{V_{sig}} = \dfrac{V_i}{V_{sig}} \cdot \dfrac{V_o}{V_i} = \dfrac{R_{in}}{R + R_{in}} \cdot A_v = \dfrac{1.6K}{100 + 1.6K} \cdot 31.75 = 29.9\ V/V$

K.P